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U.S. DEPARTMENT OF COMMERCE PATENT AND TRADEMARK OFFICE

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## TRANSMITTAL LETTER TO THE UNITED STATES

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DESIGNATED/ELECTED OFFICE (DO/EO/US)

U.S. APPLICATION NO. (IF KNOWN, SEE 37 CFR

CONCERNING A FILING UNDER 35 U.S.C. 371

09/763934

INTERNATIONAL APPLICATION NO.

PCT/FR99/02089

INTERNATIONAL FILING DATE

September 2, 1999

PRIORITY DATE CLAIMED

September 4, 1998 and Oct. 7, 1998

TITLE OF INVENTION

DIGITAL EQUALISING METHOD, AND RADIO COMMUNICATION RECEIVER IMPLEMENTING SAID METHOD

APPLICANT(S) FOR DO/EO/US

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Applicant herewith submits to the United States Designated/Elected Office (DO/EO/US) the following items and other information:

1. ☒ This is a **FIRST** submission of items concerning a filing under 35 U.S.C. 371.
2. ☐ This is a **SECOND** or **SUBSEQUENT** submission of items concerning a filing under 35 U.S.C. 371.
3. ☐ This is an express request to begin national examination procedures (35 U.S.C. 371(f)) at any time rather than delay examination until the expiration of the applicable time limit set in 35 U.S.C. 371(b) and PCT Articles 22 and 39(1).
4. ☒ A proper Demand for International Preliminary Examination was made by the 19th month from the earliest claimed priority date.
5. ☒ A copy of the International Application as filed (35 U.S.C. 371 (c) (2))
  - a. ☒ is transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☐ has been transmitted by the International Bureau.
  - c. ☐ is not required, as the application was filed in the United States Receiving Office (RO/US).
6. ☒ A translation of the International Application into English (35 U.S.C. 371(c)(2)).
7. ☒ A copy of the International Search Report (PCT/ISA/210).
8. ☐ Amendments to the claims of the International Application under PCT Article 19 (35 U.S.C. 371 (c)(3))
  - a. ☐ are transmitted herewith (required only if not transmitted by the International Bureau).
  - b. ☐ have been transmitted by the International Bureau.
  - c. ☐ have not been made; however, the time limit for making such amendments has NOT expired.
  - d. ☐ have not been made and will not be made.
9. ☐ A translation of the amendments to the claims under PCT Article 19 (35 U.S.C. 371(c)(3)).
10. ☐ An oath or declaration of the inventor(s) (35 U.S.C. 371 (c)(4)).
11. ☒ A copy of the International Preliminary Examination Report (PCT/IPEA/409).
12. ☒ A translation of the annexes to the International Preliminary Examination Report under PCT Article 36 (35 U.S.C. 371 (c)(5)).

## Items 13 to 18 below concern document(s) or information included:

13. ☐ An Information Disclosure Statement under 37 CFR 1.97 and 1.98.
14. ☐ An assignment document for recording. A separate cover sheet in compliance with 37 CFR 3.28 and 3.31 is included.
15. ☒ A **FIRST** preliminary amendment.  
A **SECOND** or **SUBSEQUENT** preliminary amendment.
16. ☐ A substitute specification.
17. ☐ A change of power of attorney and/or address letter.
18. ☒ Certificate of Mailing by Express Mail
19. ☒ Other items or information:

2 Sheets of Formal  
PCT Request Form PCT/RO/101  
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09/763934

JCO<sup>2</sup> Rec'r PCT/PTO 27 FEB 2001

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re application of DORNSTETTER, et al. )  
 )  
U.S. patent application )  
(U.S. National Phase of PCT/FR99/02089 )  
filed on September 2, 1999) )  
(corr. to French applications )  
No.98 11095 and No.98 12552) )

For : " DIGITAL EQUALISING METHOD AND RADIO COMMUNICATION  
RECEIVER IMPLEMENTING SAID METHOD "

PRELIMINARY AMENDMENT

To Honorable Commissioner of Patents and Trademarks  
Washington, D.C. 20231

S I R:

Prior to examination, please amend the above-  
identified application as follows:

IN THE SPECIFICATION:

Page 1, between lines 2 and 3, insert the heading  
-- BACKGROUND OF THE INVENTION --.

Page 5, between lines 10 and 11, insert the heading  
-- SUMMARY OF THE INVENTION --.

Page 7, delete lines 29-32.

Page 7, before line 33, insert the heading -- BRIEF  
DESCRIPTION OF THE DRAWINGS --.

Page 8, between lines 3 and 4, insert the heading  
-- DESCRIPTION OF PREFERRED EMBODIMENTS --.

IN THE CLAIMS:

Please amend Claims 1-8 as follows:

-- 1 (Amended). A digital equalization method for  
estimating discrete information symbols  $[(d_k)]$  of a  
transmitted signal] from digital samples  $[(y_k)]$  of a  
signal received over a transmission channel represented by  
a finite impulse response of  $W+1$  coefficients  
 $[(r_0, r_1, \dots, r_W)]$ ,  $W$  being an integer greater than 1,  
comprising the steps of:

- determining [the]  $W$  roots  $[(\alpha_1, \alpha_2, \dots, \alpha_W)]$  in the  
complex plane of the  $Z$ -transform  $[(R(Z))]$  of the  
impulse response;
- distributing the  $W$  roots into a first set of  $W-p$   
roots  $[(\alpha_1, \dots, \alpha_{W-p})]$  and a second set of  $p$  roots  
 $[(\alpha_{W-p+1}, \dots, \alpha_W)]$ ,  $p$  being an integer greater than 0

and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;

- obtaining an intermediate signal  $[(Y')]$  by applying a first equalization method to the received signal  $[(Y)]$  based on a finite impulse response [whose] having a Z-transform  $[(R^S(Z))$ , formed by] consisting of a polynomial of degree  $W-p$  in  $Z^{-1}$ , [has] having roots [which are] equal to the  $W-p$  roots of the first set; and
- obtaining estimations  $[(\hat{d}_k)]$  of the discrete information symbols [of the transmitted signal] by applying a second equalization method to the intermediate signal based on a finite impulse response [whose] having a Z-transform  $[(R^I(Z))$ , formed by] consisting of a polynomial of degree  $p$  in  $Z^{-1}$ , [has] having roots [which are] equal to the  $p$  roots of the second set.

2 (Amended). A method according to claim 1, wherein the first equalization method yields the intermediate signal in the form of a vector  $Y'$  of  $n+p$  samples  $[(y'_1, \dots, y'_{n+p})]$  obtained according to the relation :

$$Y' = (A'^H A')^{-1} A'^H Y$$

where  $n$  is an integer representing a frame size,  $Y$  is a vector composed of  $n+W$  samples  $[(y_k)]$  of the received signal, and  $A'$  is a matrix with  $n+W$  rows and  $n+p$  columns having a Toeplitz structure formed from the coefficients  $[(s_q)]$  of said polynomial of degree  $W-p$  in  $Z^{-1}$   $[(R^S(Z))]$ .

3 (Amended). A method according to claim 1 [or 2], wherein the second equalization method comprises implementing a Viterbi algorithm.

4 (Amended). A method according to [any one of claims 1 to 3] claim 1, wherein the unit circle distance criterion, used to distribute the  $W$  roots  $\alpha_1, \dots, \alpha_W$  of the Z-transform  $[(R(Z))]$  of the channel impulse response into the first and second sets, is expressed as a distance  $\delta_q$  of the form  $\delta_q = 1 - |\alpha_q|$  if  $|\alpha_q| \leq 1$ , and of the form  $\delta_q = 1 - 1/|\alpha_q|$  if  $|\alpha_q| > 1$ , for  $1 \leq q \leq W$ .

5 (Amended). A radio communications receiver comprising:

- conversion means [(1,3,4)] to produce digital samples  $[(y_k)]$  from a radio signal received over a transmission channel represented by a finite impulse response of  $W+1$  coefficients  $[(r_0, r_1, \dots, r_W)]$ ,  $W$  being an integer greater than 1;
- means [(6)] for measuring the channel impulse response;
- means for calculating [the]  $W$  roots  $[(\alpha_1, \alpha_2, \dots, \alpha_W)]$  in the complex plane of the Z-transform  $[(R(Z))]$  of the impulse response;
- means for distributing the  $W$  roots into a first set of  $W-p$  roots  $[(\alpha_1, \dots, \alpha_{W-p})]$  and a second set of  $p$  roots  $[(\alpha_{W-p+1}, \dots, \alpha_W)]$ ,  $p$  being an integer greater than 0 and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;

- a first equalization stage for producing an intermediate signal by applying a first equalization method to the received signal  $[(y_k)]$  based on a finite impulse response [whose] having a Z-transform  $[(R^S(Z))]$ , formed by] consisting of a polynomial of degree  $W-p$  in  $Z^{-1}$ , [has] having roots [which are] equal to the  $W-p$  roots of the first set; and
- a second equalization stage for producing estimations  $[(\hat{d}_k)]$  of the discrete symbols of a signal carried on the channel by applying a second equalization method to the intermediate signal based on a finite impulse response [whose] having a Z-transform  $[(R^I(Z))]$ , formed by] consisting of a polynomial of degree  $p$  in  $Z^{-1}$ , [has] having roots [which are] equal to the  $p$  roots of the second set.

6 (Amended). A receiver according to claim 5, wherein the first equalization stage is arranged to yield the intermediate signal in the form of a vector  $Y'$  of  $n+p$  samples  $[(y'_1, \dots, y'_{n+p})]$  obtained according to the relation:

$$Y' = (A'^H A')^{-1} A'^H Y$$

where  $n$  is an integer representing a frame size,  $Y$  is a vector composed of  $n+W$  samples  $[(y_k)]$  of the received signal, and  $A'$  is a matrix with  $n+W$  rows and  $n+p$  columns having a Toeplitz structure formed from the coefficients  $[(s_q)]$  of said polynomial of degree  $W-p$  in  $Z^{-1}$   $[(R^S(Z))]$ .

7 (Amended). A receiver according to claim 5 [or 6], wherein the second equalization stage is arranged to implement a Viterbi algorithm.

8 (Amended). A receiver according to [any one of claims 5 to 7] claim 5, wherein the means for distributing the W roots into the first and second sets make use of a unit circle distance criterion expressed as a distance  $\delta_q$  of the form  $\delta_q = 1 - |\alpha_q|$  if  $|\alpha_q| \leq 1$ , and of the form  $\delta_q = 1 - 1/|\alpha_q|$  if  $|\alpha_q| > 1$ , for  $1 \leq q \leq W$ . --



**REMARKS**

Entry of the above amendment is respectfully requested.

The amended claims 1-8 are substantially similar to original claims 1-8 as set out in the PCT International Application, with the exception that the new claims have been amended to omit multiple dependent claims and to otherwise conform to U.S. practice.

Respectfully submitted,

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DIGITAL EQUALISING METHOD  
AND RADIO COMMUNICATION RECEIVER IMPLEMENTING SAID METHOD

The invention concerns the digital equalization of signals. It has important applications in the field of radio communications.

The method is applied to the processing of a signal received from a transmitter over a channel between transmitter and receiver, whose response is known or has been estimated beforehand. One of the main issues is then that of the compromise between the performance of the equalizer and its complexity.

A complete maximum likelihood estimation of the transmitted signal is possible, for example by employing the Viterbi algorithm (see G.D. Forney Jr. : *The Viterbi Algorithm*, Proc. of the IEEE, Vol. 61, No. 3, March 1973, pages 268-278). Nevertheless, as soon as the impulse response of the channels becomes long or the number of possible discrete values becomes large, the exponentially increasing complexity of these methods renders them impracticable.

We consider the case of a radio communications channel used for the transmission of a signal composed of successive sequences or frames of  $n$  symbols,  $d_k$  ( $1 \leq k \leq n$ ). The symbols  $d_k$  have discrete values : binary ( $\pm 1$ ) in the case of a BPSK (binary phase shift keying) modulation; quaternary ( $\pm 1 \pm j$ ) in the case of a QPSK (quaternary phase shift keying) modulation.

After baseband conversion, digitization and matched filtering, a vector  $Y$  of the received signal, corresponding to the symbols transmitted over the duration of a frame, is defined by the expression :

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_k \\ \vdots \\ Y_L \end{pmatrix} = \begin{pmatrix} r_0 & 0 & 0 & \dots & 0 \\ r_1 & r_0 & 0 & & \vdots \\ & r_1 & r_0 & \ddots & 0 \\ \vdots & & r_1 & \ddots & 0 \\ & \vdots & & \ddots & r_0 \\ r_W & & & & r_1 \\ 0 & r_W & & & \vdots \\ 0 & 0 & r_W & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \dots & 0 & 0 & r_W \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_k \\ \vdots \\ d_n \end{pmatrix} + \begin{pmatrix} Y_{N,1} \\ Y_{N,2} \\ \vdots \\ Y_{N,k} \\ \vdots \\ Y_{N,L} \end{pmatrix} = A.D + Y_N \quad (1)$$

where  $W+1$  is the length in numbers of bits of the estimated impulse response of the channel,  $\underline{r} = (r_0, r_1, \dots, r_W)$  is the estimated impulse response of the channel, the  $r_q$  being complex numbers such that  $r_q=0$  for  $q<0$  or  $q>W$ ,  $y_k$  is the  $k$ -th complex sample received with  $1 \leq k \leq L=n+W$ , and  $Y_N$  is a vector of dimension  $L$  composed of samples of additive noise  $Y_{N,k}$ . The estimated impulse response  $\underline{r}$  takes into account the propagation channel, the signal shaping by the transmitter and the receiver filtering.

The matrix  $A$  of dimension  $L \times n$  has a Toeplitz-type structure, meaning that if  $\alpha_{i,j}$  denotes the term in the  $i$ -th row and in the  $j$ -th column of the matrix  $A$ , then  $\alpha_{i+1,j+1} = \alpha_{i,j}$  for  $1 \leq i \leq L-1$  and  $1 \leq j \leq n-1$ . The terms of the matrix  $A$  are given by :  $\alpha_{1,j}=0$  for  $1 < j \leq n$  ( $A$  therefore has only zeros above its principal diagonal);  $\alpha_{i,1}=0$  for  $W+1 < i \leq L$  (band-matrix structure); and  $\alpha_{i,1}=r_{i-1}$  for  $1 \leq i \leq W+1$ .

The matrix equation (1) expresses the fact that the signal received  $Y$  is an observation, with an additive noise, of the convolution product between the channel impulse response and the transmitted symbols. This convolution product can also be expressed by its Z-transform :

$$Y(Z) = R(Z) \cdot D(Z) + Y_N(Z) \quad (2)$$

where  $D(Z)$ ,  $Y(Z)$ ,  $R(Z)$  and  $Y_N(Z)$  are the Z-transforms of the transmitted symbols, of the received signal, of the impulse response and of the noise, respectively :

$$D(Z) = \sum_{k=1}^n d_k \cdot Z^{-k} \quad (3)$$

$$Y(Z) = \sum_{k=1}^L y_k \cdot Z^{-k} \quad (4)$$

$$R(Z) = \sum_{q=0}^W r_q \cdot Z^{-q} \quad (5)$$

A conventional solution to solve a system such as (1) is the so-called "zero forcing" method, by which we determine the vector  $\hat{D}_{ZF}$  of  $n$  continuous components which minimizes the quadratic error  $\varepsilon = \|A\hat{D} - Y\|^2$ . Subsequently, a discretization of the components of vector  $\hat{D}_{ZF}$  relating to each channel is performed, often through a channel decoder. The least-squares solution  $\hat{D}_{ZF}$  is given by :

$\hat{D}_{ZF} = (A^H A)^{-1} A^H Y$ , where  $A^H$  denotes the conjugate transpose of matrix  $A$ . We then are faced with the problem of inverting the Hermitian positive matrix  $A^H A$ . The inversion can be effected by various classical algorithms, either directly (method of Gauss, Cholesky, etc.) or by an iterative technique (Gauss-Seidel algorithm, gradient algorithm, etc.).

The estimation error  $D - \hat{D}_{ZF}$  is equal to  $(A^H A)^{-1} A^H Y_N$ , hence the solution includes noise with a variance:

$$\sigma^2 = E(\|D - \hat{D}_{ZF}\|^2) = N_0 \times \text{Trace}[(A^H A)^{-1}] \quad (6)$$

where  $N_0$  is the noise power spectral density. It can be

seen that noise enhancement occurs when the matrix  $A^H A$  is badly conditioned, i.e. when it has one or more eigenvalues close to zero.

5 This noise enhancement is the main drawback of the conventional solution methods. In practice, the cases where the matrix  $A^H A$  is badly conditioned are frequent, especially with multiple propagation paths.

10 A relatively simple means to partly remedy this drawback is known, by accepting residual interference in the solution, i.e. by not adopting the optimum least-squares solution, but the solution :  $D_{MMSE} = (A^H A + \hat{N}_0)^{-1} A^H Y$ , where  $\hat{N}_0$  denotes an estimation of the noise spectral density, that the receiver must then calculate. This is known as the MMSE (minimum mean square error) method and  
15 it allows the estimation variance to be reduced relative to the zero-forcing method, but introduces a bias.

The zero forcing methods and the like amount to performing an inverse filtering on the signal received by a filter which models the transfer function  $1/R(Z)$   
20 calculated using a certain approximation (quadratic in the case of zero forcing). Where one or more roots of the polynomial  $R(Z)$  (equation (5)) are situated on the unit circle, the theoretical inverse filter presents singularities rendering it impossible to estimate by a  
25 satisfactory approximation. In the case of the quadratic approximation, this corresponds to a divergence in the error variance  $\sigma^2$  when the matrix  $A^H A$  has an eigenvalue equal to zero (relation (6)).

30 This problem does not arise in methods such as the Viterbi algorithm which intrinsically take the discrete nature of the symbols into account, but which require a much higher calculating power for large-sized systems.

- 5 -

U.S. Patent 4,701,936 discloses a channel equalizer using an all-pass filter determined with reference to the Z-transform of the estimated channel impulse response.

An object of the present invention is to propose an equalization method achieving a good compromise between the reliability of the estimations and the complexity of the equalizer.

Another object is to produce an equalizer requiring a reasonable calculating power but capable of processing, with performance comparable to that of Viterbi equalizers, signals whose symbols have a relatively high number of states and/or signals carried on a channel with a relatively long impulse response.

Accordingly, the invention proposes a digital equalization method for estimating discrete information symbols from digital samples of a signal received over a transmission channel represented by a finite impulse response of  $W+1$  coefficients,  $W$  being an integer greater than 1. This method comprises the steps of:

- determining the  $W$  roots in the complex plane of the Z-transform of the impulse response;
- distributing the  $W$  roots into a first set of  $W-p$  roots and a second set of  $p$  roots,  $p$  being an integer greater than 0 and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;
- obtaining an intermediate signal by applying a first equalization method to the received signal based on a finite impulse response whose Z-transform, formed by a polynomial of degree  $W-p$  in  $Z^{-1}$ , has roots which are the  $W-p$  roots of the first set; and
- obtaining estimations of the discrete information symbols by applying a second equalization method to the intermediate signal based on a finite

impulse response whose Z-transform, formed by a polynomial of degree  $p$  in  $Z^{-1}$ , has roots which are the  $p$  roots of the second set.

The "first equalisation method" will usually be  
5 chosen so as to treat the unknown symbols as continuous variables. This leads to a process similar to an inverse filtering whose transfer function would have a form approaching the expression  $1/R^S(Z)$ , where  $R^S(Z)$  denotes the polynomial in  $Z^{-1}$  of order  $W-p$  having roots that are the  
10  $W-p$  roots furthest from the unit circle in the complex plane. In particular, it could be of the "zero-forcing" type. This process generates only a small enhancement of the noise, since the roots of the associated transfer function in  $Z$  are relatively far from the unit circle.

15 For the  $p$  roots which are closest to the unit circle, we adopt measures to avoid or to limit the incidence of the noise enhancement problem. A MMSE or similar method can be chosen as the "second equalization method". However, this second method will advantageous  
20 take into account the discrete nature of the unknown symbols. In particular, it may rely on a trellis algorithm, such as the Viterbi algorithm, whose implementation is common in channel equalizers where the system size is not too big.

25 The implementation of the second equalization method is generally more complex than the first. In each particular case, the choice of the number  $p$  allows the best compromise between the reliability of the estimations, which favors higher values of  $p$ , and the  
30 complexity of the equalizer, which favors lower values of  $p$ .

Another aspect of the present invention relates to a radio communications receiver comprising :

- conversion means to produce digital samples from a radio signal received over a transmission channel represented by a finite impulse response of  $W+1$  coefficients,  $W$  being an integer greater than 1;
- 5 - means for measuring the channel impulse response;
- means for calculating the  $W$  roots in the complex plane of the  $Z$ -transform of the impulse response;
- means for distributing the  $W$  roots into a first set of  $W-p$  roots and a second set of  $p$  roots,  $p$  being an  
10 integer greater than 0 and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;
- a first equalization stage for producing an  
15 intermediate signal by applying a first equalization method to the received signal based on a finite impulse response whose  $Z$  transform, formed by a polynomial of degree  $W-p$  in  $Z^{-1}$ , has roots which are the  $W-p$  roots of the first set; and
- 20 - a second equalization stage for producing estimations of the discrete symbols of a signal carried on the channel by applying a second equalization method to the intermediate signal based on a finite impulse response whose  $Z$  transform  
25 ( $R^I(Z)$ ), formed by a polynomial of degree  $p$  in  $Z^{-1}$ , has roots which are the  $p$  roots of the second set.

Other features and advantages of the present invention will become apparent in the following description, with reference to the attached figures, of  
30 non-limiting embodiments, in which :

- Figure 1 is a block diagram of an example of a radio communications receiver according to the invention;



- Figure 2 is a flow chart showing an embodiment of the method according to the invention; and
- Figure 3 is a diagram illustrating the performances of the method.

5       The receiver represented in Fig. 1 comprises a radio stage 1 which receives the signal picked up by antenna 2 and converts it to the baseband. The baseband signal is digitized by the analog-to-digital converter (ADC) 3 and fed to the reception filter 4. Filter 4 provides matched  
10 filtering with respect to the signal shaping by the transmitter. It delivers a digital signal at a rate of one complex sample per transmitted symbol.

      The digital signal is fed to a demodulator comprising, on the one hand, a synchronization and channel  
15 estimation module 6 and, on the other hand, an equalizer 7.

      The synchronization and channel estimation are, for example, performed conventionally by means of a synchronization sequence included by the transmitter in  
20 each signal frame. Detection of this sequence, known to the receiver, allows, on the one hand, the receiver to be synchronized relative to the temporal structure of the transmitted frames, and on the other to estimate the impulse response  $\underline{r} = (r_0, r_1, \dots, r_w)$  of the channel carrying  
25 the transmitted frames. The impulse response calculated by module 6 is fed to the equalizer 7.

      Equalizer 7 functions for example according to the flow chart shown in Fig. 2 to process each frame of the received signal, which are presented in the form of a  
30 vector  $Y = \begin{pmatrix} Y_1 \\ \vdots \\ Y_L \end{pmatrix}$ , with  $L=n+W$ , adopting the previous notation.

      The channel estimation module 6 having supplied the

W+1 complex coefficients  $r_q$  of the estimated channel impulse response, the first step 10 consists of a search of the W roots of the Z-transform of the impulse response, given by equation (5). Various conventional methods for the determination of complex roots of a polynomial can be employed at step 10. To this end, reference can be made to the work of E. DURAND : "Numerical Solutions of Algebraic Equations, Volume I, Equations of the Type  $F(x)=0$ ", Editions Masson, 1960.

10 The W roots thus found  $\alpha_1, \alpha_2, \dots, \alpha_W$  are then ordered to allow distribution into two sets, one containing the W-p roots furthest from the unit circle, and the other the p roots closest to the unit circle.

To this end, a distance  $\delta_q$  is calculated at step 11 for each of the roots  $\alpha_q$  ( $1 \leq q \leq W$ ). This distance can be conveniently obtained as follows :

$$\delta_q = \begin{cases} 1 - |\alpha_q| & \text{si } |\alpha_q| \leq 1 \\ 1 - 1/|\alpha_q| & \text{si } |\alpha_q| > 1 \end{cases} \quad (7)$$

In step 12, the roots  $\alpha_q$  of the transfer function  $R(Z)$  are sorted in order of decreasing distance:  $\delta_1 \geq \delta_2 \geq \dots \geq \delta_W$ . Then, the first W-p roots  $\alpha_1, \dots, \alpha_{W-p}$ , which are the furthest from the unit circle, are separated from the remaining roots  $\alpha_{W-p+1}, \dots, \alpha_W$ .

In step 13, the equalizer 7 develops a polynomial in  $Z^{-1}$  defined by :

$$25 \quad R^S(Z) = \prod_{q=1}^{W-p} (1 - \alpha_q \cdot Z^{-1}) = \sum_{q=0}^{W-p} s_q \cdot Z^{-q} \quad (8)$$

This allows the coefficients  $s_q$  of the transfer function  $R^S(Z)$  associated with the impulse response  $\underline{s} = (s_0, s_1, \dots, s_{W-p})$  of a virtual channel to be determined,

which corresponds to the estimated transmission channel with the contributions closest to the singularity zones removed.

We then proceed to a first equalization 14 which is effectively an inverse filtering approaching the transfer function  $1/R^S(Z)$ . Several implementations can be used to perform this inverse filtering. In particular, a zero-forcing equalization can be performed, as previously discussed. Regarding these methods, reference can be made to the work of J.G. Proakis : "Digital Communications", McGraw-Hill, 2<sup>nd</sup> Edition, 1989.

The inverse filtering 14 produces an intermediate signal in the form of a vector  $Y'$  of  $L'=n+p$  samples  $y'_1, \dots, y'_L$ . In the case of the zero-forcing method, the vector  $Y'$  is obtained from the matrix equation :

$$Y' = (A'^H A')^{-1} A'^H Y \quad (9)$$

In expression (9),  $A'$  denotes a matrix having a Toeplitz structure with  $n+W$  rows and  $n+p$  columns, generated from the coefficients  $s_q$  of the polynomial  $R^S(Z)$ :

$$A' = \begin{pmatrix} s_0 & 0 & 0 & \dots & 0 \\ s_1 & s_0 & 0 & & \vdots \\ & s_1 & s_0 & \ddots & 0 \\ \vdots & & s_1 & \ddots & 0 \\ & \vdots & & \ddots & s_0 \\ s_{W-p} & & \vdots & & s_1 \\ 0 & s_{W-p} & & & \vdots \\ 0 & 0 & s_{W-p} & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \dots & 0 & 0 & s_{W-p} \end{pmatrix} \quad (10)$$

The sorting of the roots  $\alpha_q$  results in the eigenvalues of matrix  $A'^H A'$  being relatively far removed from 0.

Alternatively, the inverse filtering can be achieved by cascading  $W-p$  filtering cells, each corresponding to

the inverse of a transfer function  $R_q^s(Z) = 1 - \alpha_q Z^{-1}$ , for  $1 \leq q \leq W-p$ . If  $|\alpha_q| = 1$ , the inverse filter of  $R_q^s(Z)$  is unfeasible. If  $|\alpha_q| < 1$ , we can develop  $1/R_q^s(Z)$  in the form :

$$5 \quad \frac{1}{R_q^s(Z)} = 1 + \alpha_q \cdot Z^{-1} + \alpha_q^2 \cdot Z^{-2} + \dots + \alpha_q^m \cdot Z^{-m} + \dots \quad (11)$$

Development (11) is causal and stable since the convergence domain contains the unit circle. The inverse filter cell can thus be generated in either a transverse or a recursive form.

10 If  $|\alpha_q| > 1$ , then  $1/R_q^s(Z)$  can be developed in the form :

$$\frac{1}{R_q^s(Z)} = -\alpha_q^{-1} \cdot Z \cdot \left( 1 + \alpha_q^{-1} \cdot Z + \alpha_q^{-2} \cdot Z^2 + \dots + \alpha_q^{-m} \cdot Z^m + \dots \right) \quad (12)$$

Development (12) is anti-causal and stable. To perform the inverse filtering cell, the development (12) is truncated and an transverse filter implementation is adopted. The anti-causality results in a delay corresponding to the length of the retained response.

We note that developments (11) and (12) justify the unit circle distance criterion  $\delta_q$  used according to relation (7).

In step 15, the equalizer 7 develops a polynomial of degree  $p$  in  $Z^{-1}$ , whose roots correspond to the  $p$  roots of  $R(Z)$  closest to the unit circle, such that  $R(Z) = R^s(Z) \cdot R^I(Z)$  :

$$25 \quad R^I(Z) = r_0 \cdot \prod_{q=W-p+1}^W (1 - \alpha_q \cdot Z^{-1}) = \sum_{q=0}^p t_q \cdot Z^{-q} \quad (13)$$

The complex coefficients  $t_q$  define the impulse response of another virtual transmission channel, whose

equalization by a zero-forcing method or the like would pose problems of noise enhancement.

The intermediate signal  $Y'$  is then subjected to an equalization according to another method, based on the impulse response  $\underline{t} = (t_0, t_1, \dots, t_p)$ . This second equalization 16 can be conveniently performed by means of a Viterbi trellis (see above-cited article from G.D. Forney Jr., or the above-cited work of J.G. Proakis).

The second equalization stage 16 generates estimations  $d_k$  of the symbols of the frame ( $1 \leq k \leq n$ ). These estimations  $d_k$  constituting the output of equalizer 7 can be input to a deinterleaving module 8 then to a channel decoder 9 which detects and/or corrects possible transmission errors.

Figure 3 illustrates the performance of the method in the case of the transmission of a signal frame according to the GSM European mobile telephone system, replacing the binary modulation of the GMSK type by an eight state phase modulation (8-PSK modulation). The impulse response was truncated at five bit time periods ( $W=4$ ). Figure 3 shows the dependence of the bit error rate (BER), expressed in %, on the ratio  $E_b/N_0$  between the energy-per-bit and the spectral density of the noise, expressed in decibels (dB). The BER is that observed in the symbol estimations after deinterleaving and channel decoding carried out by a method conforming to those employed in the GSM standard. Curve I shows the results obtained by the pure zero-forcing method, i.e. in the limit case where  $p=0$ . Curve II shows the theoretical result that would be obtained by equalizing the channel purely with the Viterbi algorithm (limit case where  $p=W$ ). In practice, the corresponding trellis should have  $8^4=4096$

states, meaning that the Viterbi equalizer could not be realized using current technology. The difference between curves I and II illustrates the superiority of the Viterbi algorithm which produces maximum likelihood estimations.

5       Curves III and IV show the results obtained by the method according to the invention, in the cases where  $p=1$  and  $p=2$ , respectively. The quite significant improvement already achieved for the value  $p=1$  with respect to pure zero forcing can be seen.

10       By way of an example, equalization of a GSM signal frame by a pure Viterbi algorithm, under the conditions of Figure 3, would require of the order of 8.45 million floating-point operations, i.e. around 1.83 Gflops, while the implementation of the present invention under the same  
15       conditions requires of the order of 19,000 floating-point operations ( $\approx 4.2$  Mflops) in the case of  $p=1$ , including searching the roots of  $R(Z)$  and  $1/R^S(Z)$  inverse filtering by the zero-forcing method. The number increases to the order of 129,000 operations ( $\approx 28$  Mflops) in the case of  
20        $p=2$ , which remains compatible with the capacity of currently available digital signal processors (DSP).

C L A I M S

1. A digital equalization method for estimating discrete information symbols ( $d_k$ ) from digital samples ( $y_k$ ) of a signal received over a transmission channel represented by a finite impulse response of  $W+1$  coefficients ( $r_0, r_1, \dots, r_W$ ),  $W$  being an integer greater than 1, comprising the step of determining the  $W$  roots ( $\alpha_1, \alpha_2, \dots, \alpha_W$ ) in the complex plane of the Z-transform ( $R(Z)$ ) of the impulse response, characterized in that it further comprises the steps of:

- distributing the  $W$  roots into a first set of  $W-p$  roots ( $\alpha_1, \dots, \alpha_{W-p}$ ) and a second set of  $p$  roots ( $\alpha_{W-p+1}, \dots, \alpha_W$ ),  $p$  being an integer greater than 0 and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;
- obtaining an intermediate signal ( $Y'$ ) by applying a first equalization method to the received signal ( $Y$ ) based on a finite impulse response whose Z-transform ( $R^S(Z)$ ), formed by a polynomial of degree  $W-p$  in  $Z^{-1}$ , has roots which are the  $W-p$  roots of the first set; and
- obtaining estimations ( $\hat{d}_k$ ) of the discrete information symbols by applying a second equalization method to the intermediate signal based on a finite impulse response whose Z-transform ( $R^I(Z)$ ), formed by a polynomial of degree  $p$  in  $Z^{-1}$ , has roots which are the  $p$  roots of the second set.

2. A method according to claim 1, wherein the first equalization method yields the intermediate signal in the form of a vector  $Y'$  of  $n+p$  samples  $(y'_1, \dots, y'_{n+p})$  obtained according to the relation :

$$Y' = (A'^H A')^{-1} A'^H Y$$

where  $n$  is an integer representing a frame size,  $Y$  is a vector composed of  $n+W$  samples  $(y_k)$  of the received signal, and  $A'$  is a matrix with  $n+W$  rows and  $n+p$  columns having a Toeplitz structure formed from the coefficients  $(s_q)$  of said polynomial of degree  $W-p$  in  $Z^{-1}$  ( $R^S(Z)$ ).

3. A method according to claim 1 or 2, wherein the second equalization method comprises implementing a Viterbi algorithm.

4. A method according to any one of claims 1 to 3, wherein the unit circle distance criterion, used to distribute the  $W$  roots  $\alpha_1, \dots, \alpha_W$  of the  $Z$ -transform ( $R(Z)$ ) of the channel impulse response into the first and second sets, is expressed as a distance  $\delta_q$  of the form  $\delta_q = 1 - |\alpha_q|$  if  $|\alpha_q| \leq 1$ , and of the form  $\delta_q = 1 - 1/|\alpha_q|$  if  $|\alpha_q| > 1$ , for  $1 \leq q \leq W$ .

5. A radio communications receiver comprising :

- conversion means (1,3,4) to produce digital samples  $(y_k)$  from a radio signal received over a transmission channel represented by a finite impulse response of  $W+1$  coefficients  $(r_0, r_1, \dots, r_W)$ ,  $W$  being an integer greater than 1;
- means (6) for measuring the channel impulse response;



- means for calculating the  $W$  roots  $(\alpha_1, \alpha_2, \dots, \alpha_W)$  in the complex plane of the  $Z$ -transform  $(R(Z))$  of the impulse response;
- means for distributing the  $W$  roots into a first set of  $W-p$  roots  $(\alpha_1, \dots, \alpha_{W-p})$  and a second set of  $p$  roots  $(\alpha_{W-p+1}, \dots, \alpha_W)$ ,  $p$  being an integer greater than 0 and smaller than  $W$ , the roots of the second set being closer to the unit circle than those of the first set according to a determined distance criterion in the complex plane;
- a first equalization stage for producing an intermediate signal by applying a first equalization method to the received signal  $(y_k)$  based on a finite impulse response whose  $Z$  transform  $(R^S(Z))$ , formed by a polynomial of degree  $W-p$  in  $Z^{-1}$ , has roots which are the  $W-p$  roots of the first set; and
- a second equalization stage for producing estimations  $(\hat{d}_k)$  of the discrete symbols of a signal carried on the channel by applying a second equalization method to the intermediate signal based on a finite impulse response whose  $Z$  transform  $(R^I(Z))$ , formed by a polynomial of degree  $p$  in  $Z^{-1}$ , has roots which are the  $p$  roots of the second set.

6. A receiver according to claim 5, wherein the first equalization stage is arranged to yield the intermediate signal in the form of a vector  $Y'$  of  $n+p$  samples  $(Y'_1, \dots, Y'_{n+p})$  obtained according to the relation :

$$Y' = (A'^H A')^{-1} A'^H Y$$

where  $n$  is an integer representing a frame size,  $Y$  is a vector composed of  $n+W$  samples  $(y_k)$  of the received signal, and  $A'$  is a matrix with  $n+W$  rows and  $n+p$  columns

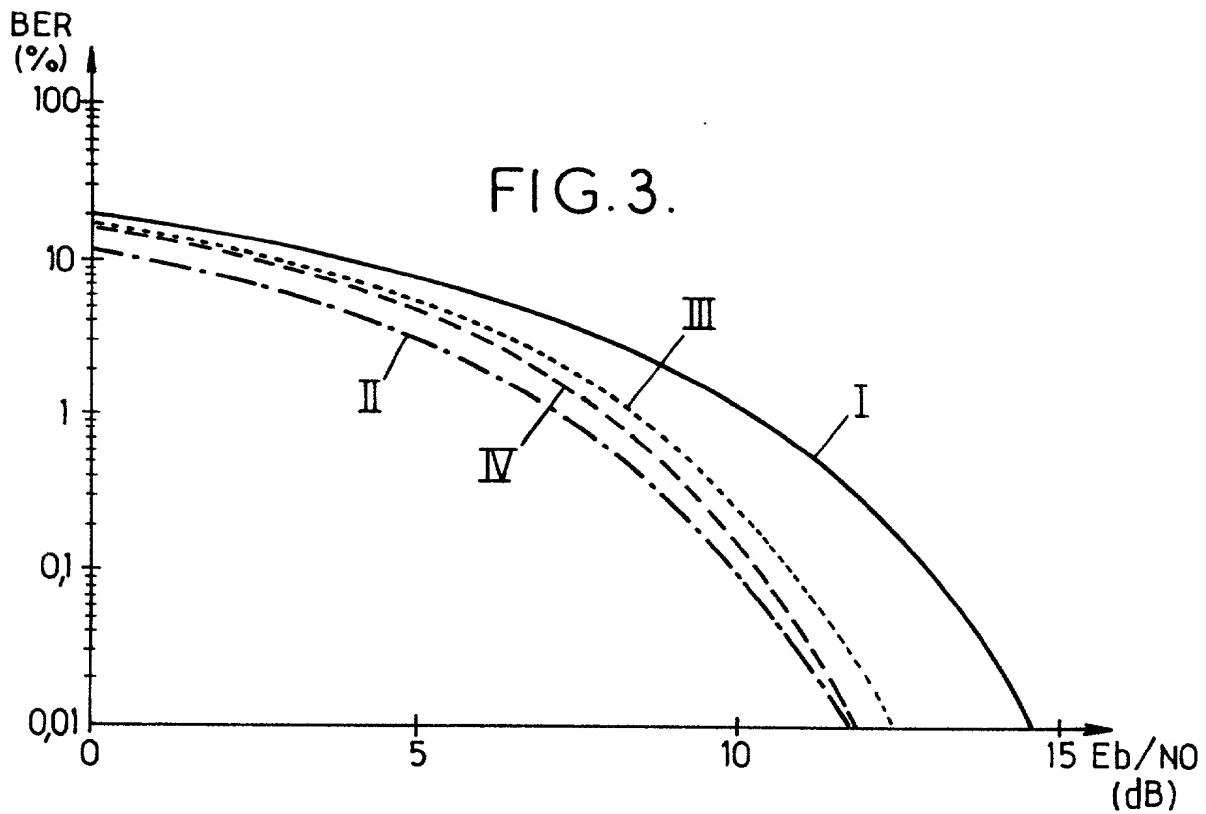
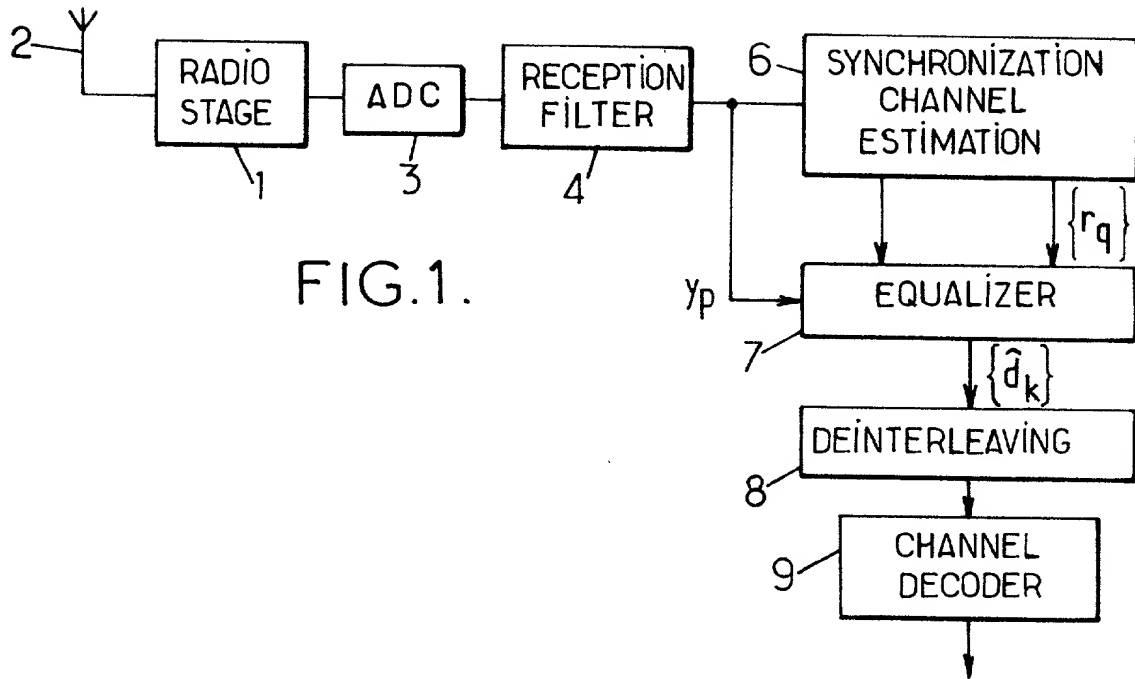
having a Toeplitz structure formed from the coefficients  $(s_q)$  of said polynomial of degree  $W-p$  in  $Z^{-1}$  ( $R^S(Z)$ ).

7. A receiver according to claim 5 or 6, wherein the second equalization stage is arranged to implement a Viterbi algorithm.

8. A receiver according to any one of claims 5 to 7, wherein the means for distributing the  $W$  roots into the first and second sets make use of a unit circle distance criterion expressed as a distance  $\delta_q$  of the form

$\delta_q = 1 - |\alpha_q|$  if  $|\alpha_q| \leq 1$ , and of the form  $\delta_q = 1 - 1/|\alpha_q|$  if  $|\alpha_q| > 1$ , for  $1 \leq q \leq W$ .

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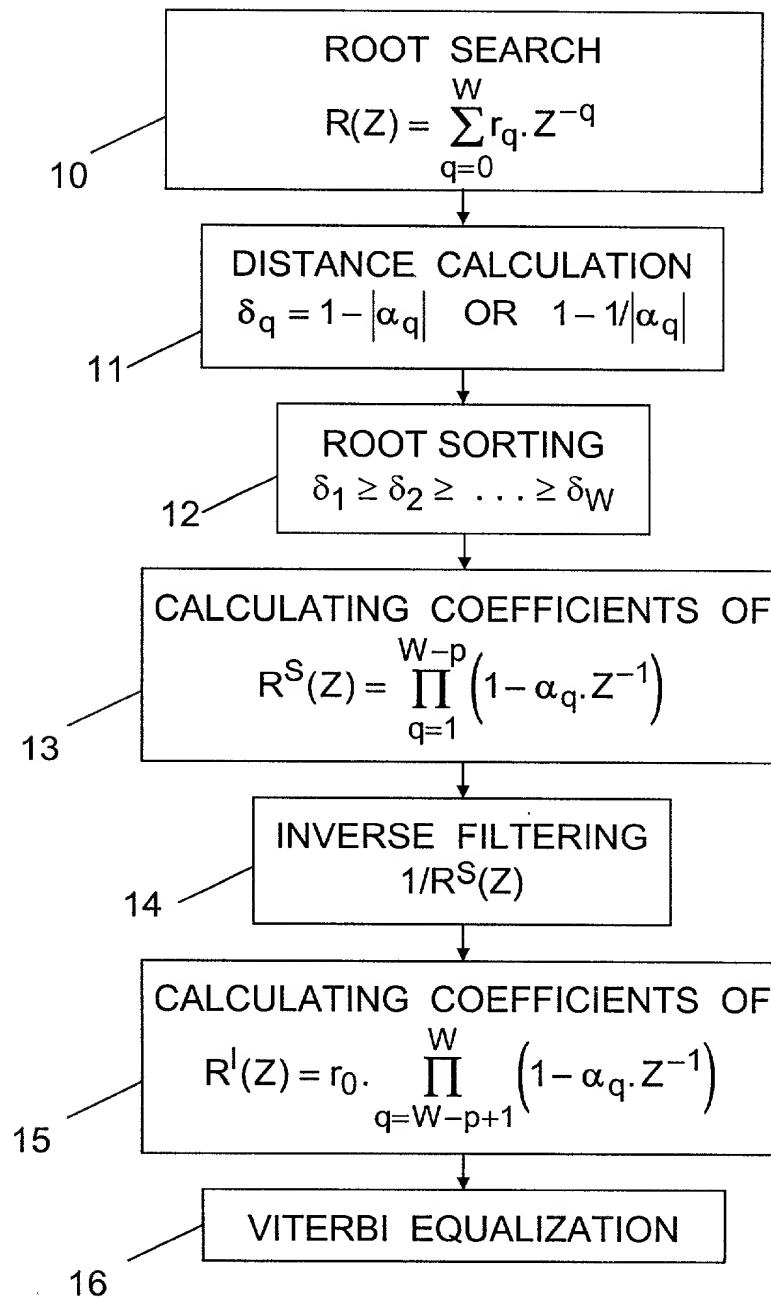


FIG.2.

Docket No.

# Declaration and Power of Attorney For Patent Application

## English Language Declaration

As a below named inventor, I hereby declare that:

My residence, post office address and citizenship are as stated below next to my name,

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled **DIGITAL EQUALISING METHOD AND RADIO COMMUNICATION RECEIVER IMPLEMENTING SAID METHOD**

the specification of which

(check one)

☒ is attached hereto.

☐ was filed on \_\_\_\_\_ as United States Application No. or PCT International

Application Number \_\_\_\_\_

and was amended on \_\_\_\_\_

(if applicable)

I hereby state that I have reviewed and understand the contents of the above identified specification, including the claims, as amended by any amendment referred to above.

I acknowledge the duty to disclose to the United States Patent and Trademark Office all information known to me to be material to patentability as defined in Title 37, Code of Federal Regulations, Section 1.56.

I hereby claim foreign priority benefits under Title 35, United States Code, Section 119(a)-(d) or Section 365(b) of any foreign application(s) for patent or inventor's certificate, or Section 365(a) of any PCT International application which designated at least one country other than the United States, listed below and have also identified below, by checking the box, any foreign application for patent or inventor's certificate or PCT International application having a filing date before that of the application on which priority is claimed.

Prior Foreign Application(s)

Priority Not Claimed

98 11095	FRANCE	04-09-1998	<input type="checkbox"/>
(Number)	(Country)	(Day/Month/Year Filed)	
98 12552	FRANCE	07-10-1998	<input type="checkbox"/>
(Number)	(Country)	(Day/Month/Year Filed)	
			<input type="checkbox"/>
(Number)	(Country)	(Day/Month/Year Filed)	

2 Dec 98

I hereby claim the benefit under 35 U.S.C. Section 119(e) of any United States provisional application(s) listed below:

\_\_\_\_\_  
(Application Serial No.)

\_\_\_\_\_  
(Filing Date)

\_\_\_\_\_  
(Application Serial No.)

\_\_\_\_\_  
(Filing Date)

\_\_\_\_\_  
(Application Serial No.)

\_\_\_\_\_  
(Filing Date)

I hereby claim the benefit under 35 U. S. C. Section 120 of any United States application(s), or Section 365(c) of any PCT International application designating the United States, listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States or PCT International application in the manner provided by the first paragraph of 35 U.S.C. Section 112. I acknowledge the duty to disclose to the United States Patent and Trademark Office all information known to me to be material to patentability as defined in Title 37, C. F. R., Section 1.56 which became available between the filing date of the prior application and the national or PCT International filing date of this application:

\_\_\_\_\_  
(Application Serial No.)

\_\_\_\_\_  
(Filing Date)

\_\_\_\_\_  
(Status)  
(patented, pending, abandoned)

\_\_\_\_\_  
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(Application Serial No.)

\_\_\_\_\_  
(Filing Date)

\_\_\_\_\_  
(Status)  
(patented, pending, abandoned)

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.

POWER OF ATTORNEY: As a named inventor, I hereby appoint the following attorney(s) and/or agent(s) to prosecute this application and transact all business in the Patent and Trademark Office connected therewith. (list name and registration number)

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Mary Spalding Burns	32,116
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Fourth inventor's signature	Date
Residence	
Citizenship	
Post Office Address	

Full name of fifth inventor, if any	
Fifth inventor's signature	Date
Residence	
Citizenship	
Post Office Address	

Full name of sixth inventor, if any	
Sixth inventor's signature	Date
Residence	
Citizenship	
Post Office Address	



Docket No.

# Declaration and Power of Attorney For Patent Application

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My residence, post office address and citizenship are as stated below next to my name,

I believe I am the original, first and sole inventor (if only one name is listed below) or an original, first and joint inventor (if plural names are listed below) of the subject matter which is claimed and for which a patent is sought on the invention entitled *Digital Equalizing method and radio communication receiver implementing said method.*

the specification of which

(check one)

☒ is attached hereto.

☐ was filed on \_\_\_\_\_ as United States Application No. or PCT International Application Number \_\_\_\_\_ and was amended on \_\_\_\_\_ (if applicable)

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Full name of sole or first inventor	Jean-Louis DORNSTETTER
Sole or first inventor's signature	<i>[Signature]</i> Date
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Fourth inventor's signature	Date
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Citizenship	
Post Office Address	

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Citizenship	
Post Office Address	

Full name of sixth inventor, if any	
Sixth inventor's signature	Date
Residence	
Citizenship	
Post Office Address	